1. Reflect on one research article.

In her article “Problem solving and at-risk students: Making ‘mathematics for all’ a classroom reality” Margot Fulton Robert shares her experience of transitioning from an affluent suburban school into a high-poverty rural school. She describes how previously successful teaching strategies did not work anymore in her new school and how she had to re-think her teaching strategies. The reader learns about the instructional changes Robert Fulton made and what approaches worked well to get at-risk students engaged in the Math classroom.

In their book “Effectief leren” (effective learning) the two Dutch educational consultants Sebo Ebbens and Simon Ettekoven explain six key concepts that have to be present in a lesson to make it effective. Each of those key concepts can be found in Margot Fulton Robert’s narrative article and this is the reason why the content is very useful for practicing as well as newly qualified teachers. The first key concept is a well-structured lesson. This concept is not explicitly discussed but the way Fulton Robert describes her thinking about the lesson planning and her ability to adjust her practices, and when she consults educational literature it lets the reader conclude that she does pay careful attention to her lesson planning and structure. The second key concept is the appropriate level of the content that students ought to learn. When Fulton Robert realizes that her new students to not possess the endurance, expertise and metacognitive skills to be successful in her math classroom she adjusts the tasks. She says “I may give up on the activity but not on my students” indicating that she rather adjusts the task to let students experience success than thinking that her students are unable to do Math. The third concept is that learning has to be meaningful. This concept does not only mean that mathematical problem should be realistic; it is more general and can also be applied to sub-goals that are taught. In Fulton Robert’s case that means that she teaches the children skills they need to solve the magic square. She could also hand them out sheets with

rows of arithmetic exercises, but such a task will have no meaning for the students and be perceived as dull and boring. The fourth concept is individual accountability. When the students learn, step by step, how to solve a magic square they are also required to come to the front of class and teach their solutions to the other students. This is a technique to hold a student accountable, encourage their participation level and engagement with the task. This very same technique also makes student's thinking and learning visible, which is the fifth key concept. The visibility of student's thinking also increases, when they are trained to share their strategies and their teacher starts to use journals with the class. The sixth and last key concept is motivation. Many teachers think that a student's motivation is inherent, fixed and unchangeable and do not realize how much their lesson and their behaviour influences a student's motivation. Margot Fulton Robert's article is exemplary for how good instruction can motivate students. When her students engage in "self-protective disruptions" she realises that they do so because their self-confidence is challenged (Robert, 2002) and not because of an inherent character trait that makes them rambunctious. To motivate her students she understands that they need to experience success and that "confidence grows largely as a result of experience" (Robert, 2002). When she gives students a chance to "work together, share strategies, get frequent hands on experiences" with initially a lot of teacher guidance, the students also receive plenty of feedback about their own learning. Instant feedback is very important to motivate students, as it builds their confidence to move on and tackle more complex tasks. Margot Fulton Robert introduces a teacher-student dialogue journal that serves two purposes. It makes a student's learning meaningful because it acknowledges the student's effort and thinking. At the same time it conveys to a student that the teacher is genuinely interested in what they do, what they think and what they learned or didn't. The article is not only useful because the content reflects good

teaching practise, it is also relevant because it contains a lot of practical teaching tips that can immediately influence instruction.

The most important statement is that a teacher can give up on an activity but not on the students. When students become ebullient, disruptive and threaten the teacher Fulton Robert's analysis is spot on: they are angry because the teacher makes them feel incompetent (Robert, 2002). This is a lesson every teacher can take back to her own classroom and use it to reflect on lessons that went awfully wrong. Instead of putting the blame on unwilling students, the objectives have to be examined and the teacher has to decide whether they may have to be adjusted. From the article the reader also learns that student's do not learn how to think by simply doing activities. This is a valuable insight when a teacher works with at-risk students who come to school less prepared than their affluent and already well educated counterparts. Most students however can learn how to think given that the teacher takes the time to teach strategies and model various thinking processes. The students in this fifth grade class-room also learn that there is more than one strategy to solve a problem, a teaching philosophy that should be fully endorsed by every mathematics teacher and a simple one to implement. Often teachers assess students only with exams, the state mandated tests and whatever grading they are required to do. Margot Fulton Roberts presents a few useful ideas on how to assess students besides the mandated assessments. Self-assessment is a teaching tool that is easy to implement, even when students seem to be inept at first. Over time it will help them to develop metacognitive skills and they will become adept at reflecting on their learning and thinking. On a final note, the statement that "Once we say that some children are not capable thinkers [...], we have all but guaranteed that they will not be so" is a crucial thought that should affect every teacher's instruction. Any teacher who has no expectation of her students cannot expect them to perform well. Students, however young they

are, do have an excellent sense of what the expectations are and most times to live up to them, be they positive or negative.

2. The essential components of an effective Mathematics lesson.

An effective Mathematics lesson consists of several different components which range from those that are more general in nature and can be applied to all kind of subjects taught to those that are specific because they have a concrete matter related to the mathematics classroom.

The very first component is one that must be clear even prior to planning and certainly prior to teaching namely a **well-formulated objective**. The objective is the core of any lesson and before planning starts it must be clear what goal(s) should be achieved. In the state of Virginia the Standards of Learning (SOL) provide the objectives that the teacher has to ensure are met. Besides the SOL that is chosen for the lesson, or sometimes dictated by school authorities, the teacher must still formulate a concrete goal that describes what exactly the student will have learned after the lesson.

The lesson's goals and content must be on an **appropriate level** so students can achieve success. When the content of a lesson is either to advanced or too rudimentary students will become frustrated and lose motivation. Students who cannot, over a prolonged period of time, achieve the goals will lose their confidence and not be willing to engage in challenging tasks at all. Eventually this may lead to a real Math anxiety and the conviction that the student lacks the inherent ability to learn and do Math. Hence an appropriate level of goals or content must be a component of any effective Mathematics classroom. A mathematical theory that supports this component is the van Hiele Levels of Geometric thought. Learners have to pass through all of the four stages and cannot learn something that is beyond the stage they are in. Tasks given to students should enable them to practice skills that are appropriate for their van Hiele level and help them to

prepare for the next level of geometric thought. A learner who has not reached the level 2 of Informal Deduction cannot perform a task that is on the 3rd level of Deduction. On a concrete level this means that a student who just begins "to think about properties of geometric objects without the constraints of a particular object" cannot "work with abstract statements about geometric properties and make conclusions based on logic". (van de Walle, 2010)

During the planning of a lesson the teacher must also take student's background, abilities and prior knowledge into account. In general this is called **accommodation**. Students have different learning styles, come from various backgrounds and have different needs. To ensure equity in the mathematics classroom all those must be accommodated.

Assessments are a crucial component of the mathematics lesson. Assessments have to be on-going and varied. In an effective mathematics lesson a teacher uses formal and informal assessment techniques to make inferences about student's thinking. This will enable the teacher to make appropriate instructional decisions so the learning goals can be met by the students. A pre-assessment is necessary to determine the level and background knowledge of students. Therefore assessments should not only consist of exams and standardized tests but also include alternative assessments. Such are projects, student journals, or short ones like entrance/exit slips.

These assessments result in the component of on-going **feedback**. Students need to be given feedback frequently and in a timely manner. A good way to ensure that feedback incorporates learning is to grade student's work in a timely manner and then give students a chance to correct their own mistakes and let them formulate how they learned from their mistakes. This can be done for instance by allowing students to correct their mistakes and hand in the corrected work to receive extra credit. An approach like this

will make test correction and feedback meaningful and help to improve a student's learning. On a more informal level, feedback can also be given in verbal form during classroom discussions or one-on-one conversations. Students do want to know if they are on the right track and will be motivated when they know that they are. It is important though to know that giving the right answer to a student does not equal giving feedback.

During an effective mathematics lesson a meaningful **classroom discourse** is important. This component however does more than just provide feedback. A good discourse "provides a foundation for creating authentic classroom discussions" (Rawding and Wills, 2012). During a good classroom discussion students learn how to talk about their mathematical thinking and explain their reasoning. A mathematics teacher can help students to develop their metacognitive skills and expose their thinking by asking the right questions. Creating thoughtful, provocative questions is a challenge for many teachers (Rawding and Wills, 2012). But the more a teacher practises this the better she will become. There are several different levels of questioning and not all will lead to an effective discourse. Closed questions are directed at recalling facts or either answering 'yes' or 'no'. Questions that will lead to the exposure of student's thinking and stimulate the student's learning must be open ended. Open ended questions demand that students explain their thinking process, their reasoning, contrast and compare or transfer their knowledge to a new situation. Open ended questions are however not the acme of good classroom discussions. The sub-component that must be acknowledged is teacher wait time. Asking a meaningful open-ended question is lost when the teacher does not give students enough time to think. It is very easy to fall into the trap of asking a question and then answering it yourself after just two seconds. Increasing wait time, however difficult at first, will also yield qualitative higher answers.

Meaningful tasks should be an integral component of every Math lesson. Mathematical tasks become meaningful when they full-fill the following criteria: a student must be able to relate to the task and be able to connect it to something she is already familiar with (Crawford and Witte, 1999). This can be something that has been learned previously, or the student's life experience. When students can experience mathematical tasks by investigating, discovering or exploring they do become meaningful. Oftentimes students see no meaning when they cannot understand how they will put their newly acquainted skills to use in real life. Students who are given task that allows them to put concepts to use will see the task as meaningful (Crawford and Witte, 1999).

To ensure that a mathematical lesson is effective students must also be presented with a **variety of tasks**. Students who ought to develop concepts must be given the chance to do so numerically, graphically, symbolically as well as verbally. When a teacher accommodates the needs of different learners this "Rule of Four" should already have been used but it can be enhanced when besides the explanations the mathematical tasks also follow this concept. This component also relates to the previous component because it allows students to explore, investigate and discover mathematical concepts in a meaningful way. Using a variety of tasks ensures and investigative approach which is related to constructivism and has been proven to be the most effective teaching method.

Employing the two previously mentioned components will lead students to develop a **variety of strategies**. A component of an effective mathematical lesson is to allow students to use a variety of strategies and alternative algorithms rather than teaching a procedure and insisting that there is only one correct way to arrive at one correct solution. This is another component that is tied to accommodation. As learners have different needs they should be presented different options to reduce error and increase conceptual

understanding (Randolph and Sherman). Students who have more than one option will also increase their critical thinking skills and become more skilled with arithmetic operations (Randolph and Sherman). A lot of teachers have not had this option when they themselves were taught mathematics and it can at first be daunting to put this practice into place in one's own classroom. A reason is also that it is much less time consuming for the teacher when only one procedure is taught, practiced and assessed. This is however false logic because students who are taught in such a way may fail at a later stage.

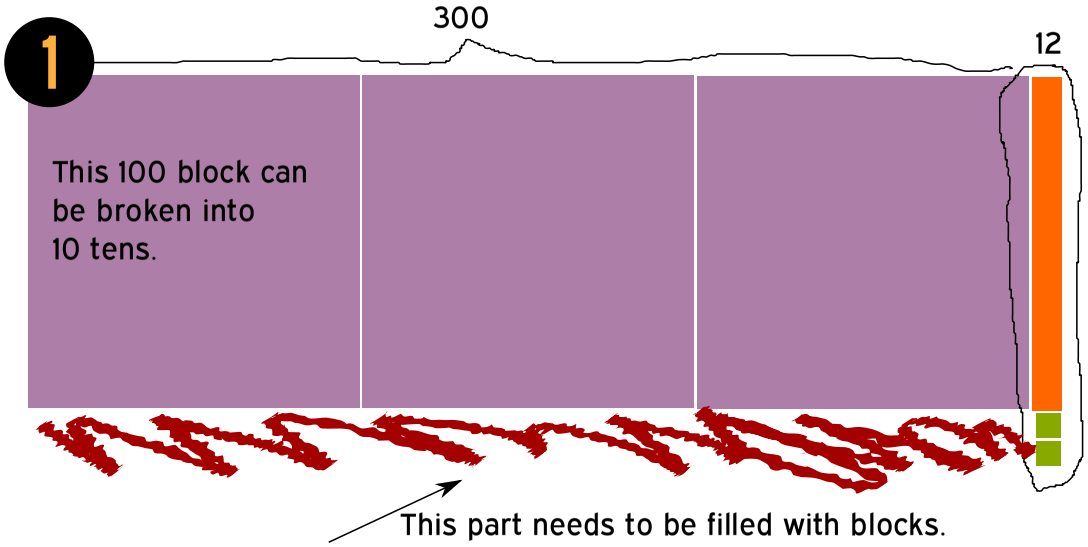
A very tangible and concrete component of an effective mathematical lesson is the use of **manipulatives**. This component relates to the components of feedback, variety of tasks, meaningful tasks and variety of strategies. The use of manipulatives serves the investigative approach and while students use them they can provide instant feedback for the results students get. As students develop problem solving skills they are not confronted with feedback that tells them they are either right or wrong, but that they have to go a step further or re-think their strategy to arrive at a solution. This makes the task meaningful. When students use concrete or virtual manipulatives they have the chance to develop individual strategies, build mathematical concepts and knowledge retention will increase.

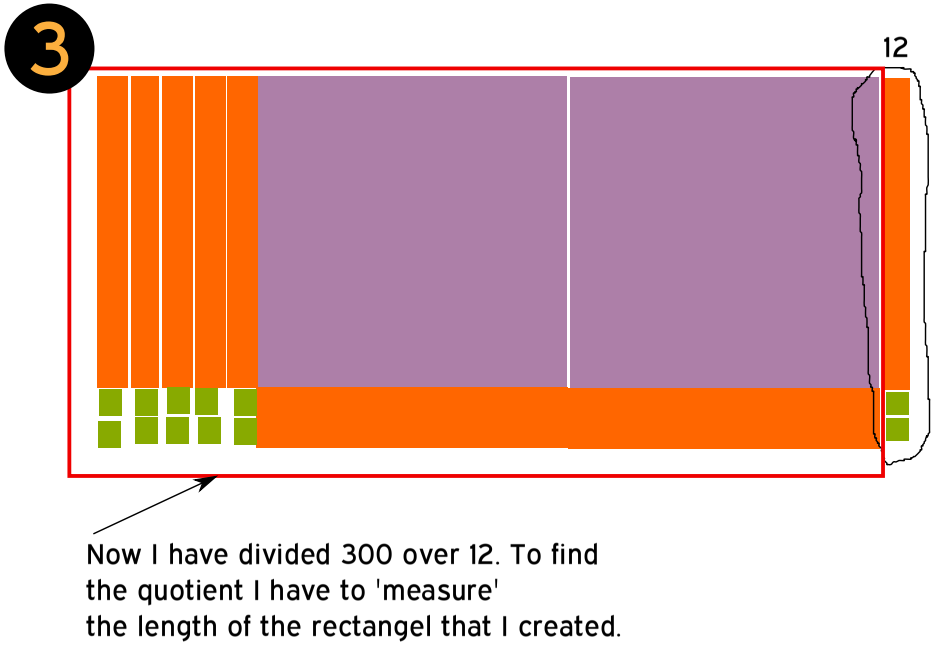
All of these components intersect. They cannot be used in singularity to ensure student's success. Instead each component resembles a cog in the machinery of the effective mathematic lesson. To ensure a positive outcome in student's learning all these cogs have to fit, be adjusted and used in such a way that the machinery is working and runs smoothly.

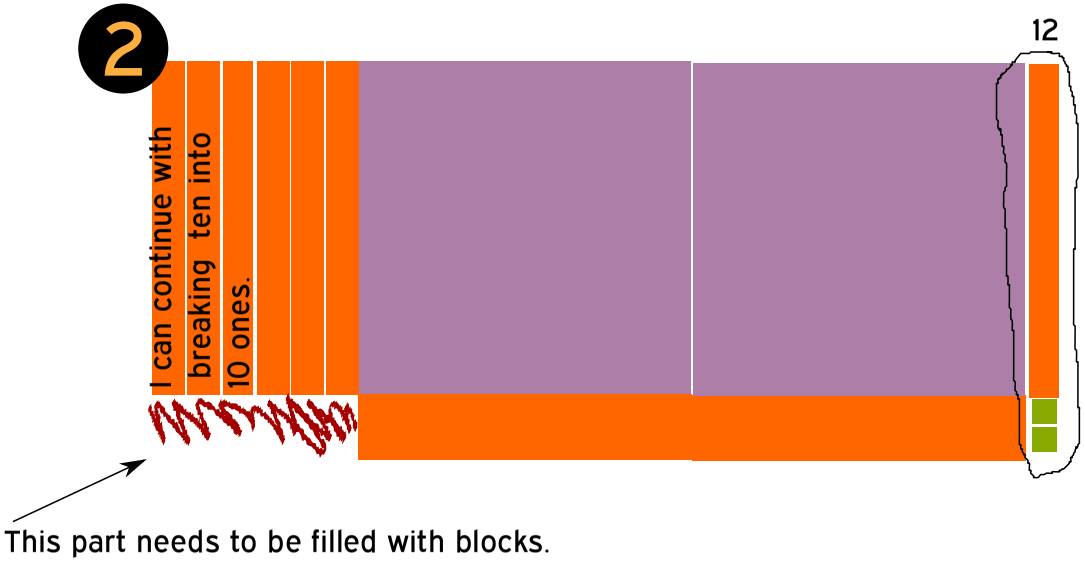
3. The use of base-ten blocks in developing one concept

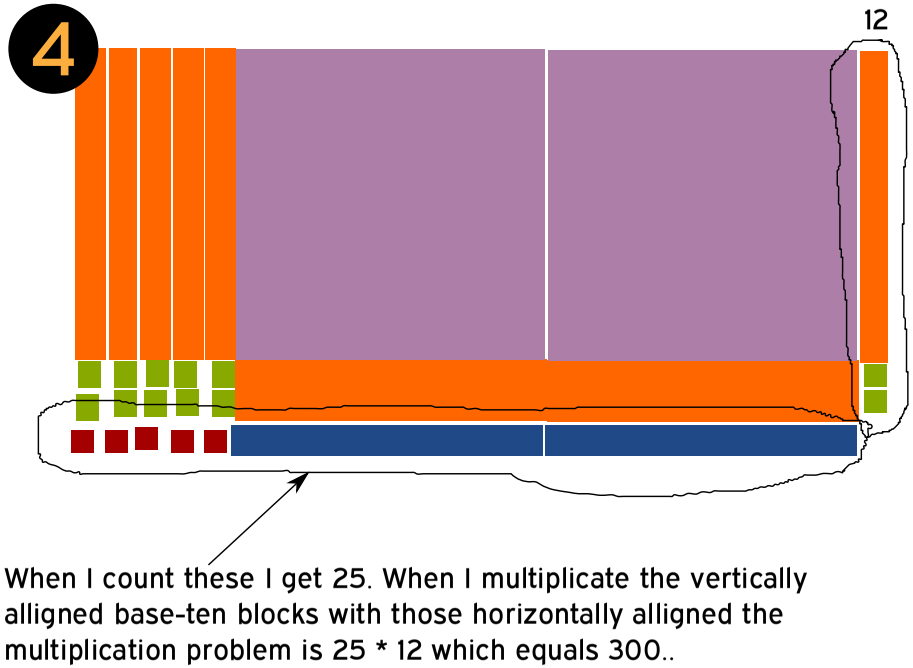
Students sometimes struggle with division and the use of base-ten blocks can help to ensure that students learn what concept underlies division. Students who have used the base-ten blocks for multiplication will already be familiar with creating the rectangle to fill in the space between to numbers laid out in base-ten blocks, one horizontally and the other one vertically. In the case of multiplication the student has to count the number of blocks used to fill in the space. Division is the opposite of multiplication and with base-ten blocks that becomes visible. With the use of base-ten blocks the student can learn that numbers can be broken apart and redistributed in different ways. In the first example the dividend is re-arranged and re-grouped to fit the divisor. The second example represents a form similar to repeated addition because multiple groups of 12 are made and then re-grouped to see if the amount of the dividend is reached. When students are working with base-ten blocks they can work on both of these concepts which are in fact simply two different interpretation of division. In the first example the question is how many there are in each group because 300 objects are equally divided over 4. In the second example the question is how many groups of twelve will make 300. This leads to the conclusion that the use of base-ten blocks lets students discover and explore the two different ways a division problem can be interpreted.

Example 1: 300 / 12



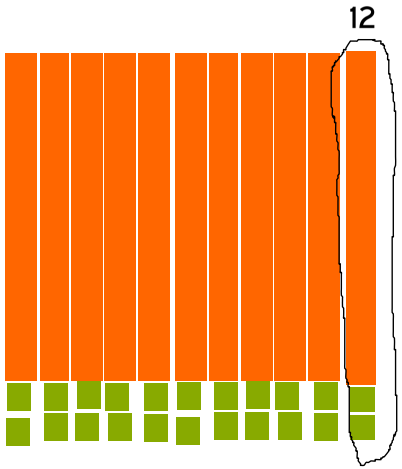




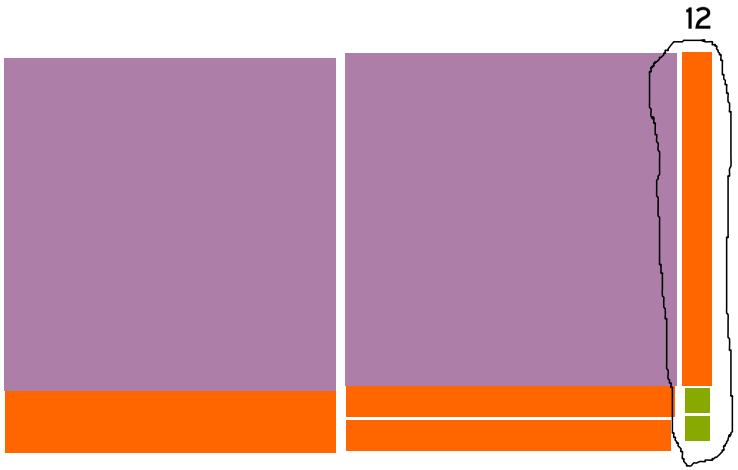


Example 2: 300 / 12

1 I start with lining up groups of twelve.
Once I lined up 10 groups of twelve
I realise that I can exchange the tens for
a hundred block and the ones for 2 tens.



2 Immediately I
see that I can
add the same
amount and still
haven't reached
three hundred. I
will add
a few more sets
of twelve.



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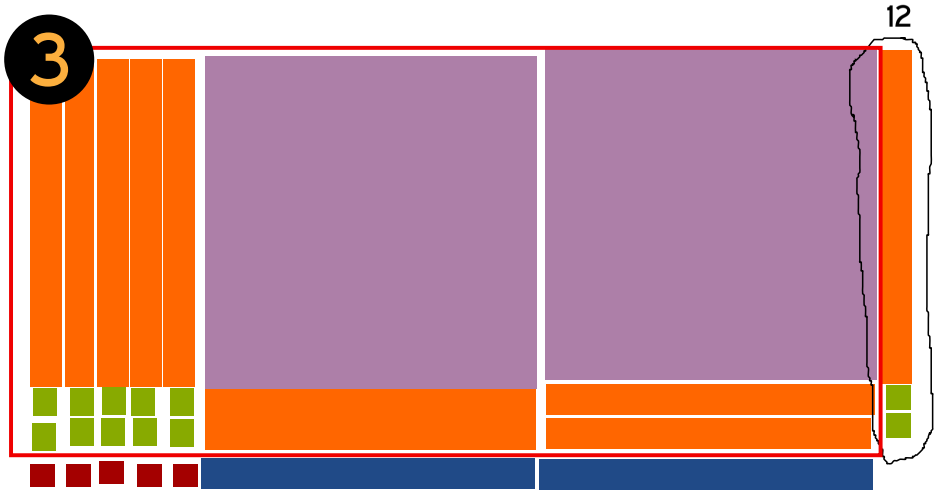

Now I have added 5 more groups of twelve.
When I count my blocks they add up to 300.
Counting the rows, I see that there are 25.

4. Explain what it means to say students must "do mathematics"

When student's "do mathematics" they are actively engaged with mathematics. Students are engaged when they are taught Math using the investigative approach. Like the term suggests, students will investigate and discover concepts themselves. They will develop higher-order thinking skills and be able to work and explore math topics in a group where every single group member contributes. Students will manipulate materials to solve problems and develop problem-solving skills and be able to verbalize their reasoning.

A task we did in class that is an example of 'doing mathematics' was the question about the Volunteers. The task consisted of a description about a group of students who had volunteered for different purposes in their community. The question was how many students were surveyed.

To solve this problem, a student could be taught set theory and the strategies of using Venn diagrams to find out the total number of students. When a student is not taught this procedure in advance the student has to engage and do mathematics. This includes finding a strategy how to solve this problem, analysing the information that is given and how it can be used to find the answer. Using a trial & error method does not work because the student was also asked to explain the reasoning. In class student came up with very different approaches, from using Venn diagrams to solving the problem with tables and groupings.

5. Give your overall review of the relevance of the requirements of this course.

In my opinion all the requirements for this course were relevant and useful. The mathematical components of the weekly reflection were related to the topics and content we had worked on in class and solving problems on your own helped to develop a different and better understanding of concepts and problem-solving strategies. The children's literature presentation were relevant because literature is a great way to make Math connections and not knowing too much English children's literature it was helpful to find more relevant books. I also learned about many interesting activities in connection with literature by watching my class-mates presentations. Naturally the unit lesson plan was the assignment that I found most relevant to the future practice of teaching. It was particularly useful because as a student I can devote much more time to create a detailed lesson plan and think about it in depth. I know how hard pressed for time teachers are once one teaches full-time and it is a great opportunity to create and collect resources as well as to internalize the important components of a good mathematics lesson. Presenting an activity to class was helpful to find out how students will react to the task and it gave me a better insight in how to organise the task and what I would need to pay attention to would I teach this to young students. The strand study was helpful to get an oversight about the development of Math concepts across the grades but I could have done without the group component. Unfortunately I found the minimalist attitude of other students contagious and noticed how it hampered my initial enthusiasm for the task.

6. Give your overall review of the classroom activities during this course.

The classroom activities were one of the main reasons why I had decided to sign up for this class. In the previous course the classroom activities had also been interesting and I have developed a small collection of great math activities that I can later use in my own classroom. The activities were valuable because they can be adjusted, modified and many materials will either be present in the classroom or can be relatively easy reproduced. Overall I'm glad I signed up for this course and do think I learned a lot during this summer semester. It was mainly the very practical approach and being introduced to hands-on teaching methods that I will be able to remember and use in the future.